Understanding Correlated Error Events in Quantum Computers

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for Undergraduates

Abstract

Quantum Computing is at a critical juncture. Prototype quantum computers are now at a level of reliability and scale where researchers can run small scale algorithms. However, modeling errors in near-term Noisy Intermediate-Scale Quantum (NISQ) devices is necessary to harness their full potential. Existing frameworks such as IBM Qiskit are limited in their capacity to model and simulate complex noise events. We study the use of Probabilistic Graphical Models (PGMs) as a natural abstraction to efficiently simulate and model quantum circuits with no noise, uncorrelated noise (bit-flip and amplitude damping) and correlated noise using complex-valued Bayesian networks. By definition, Bayesian networks are a type of PGM that articulate conditional dependencies between events through the use of Directed Acyclic Graphs (DAGs). We study well-known quantum algorithms such as Deutsch-Jozsa and Simon's algorithms, and transform their circuit representations into Bayesian network models by extending Python's pgmpy library to allow for complex-valued networks. By using exact inference algorithms like Variable Elimination, we are able to create valuable inferences that can be converted into density matrices using our extension of pgmpy. We validate that our models produce correct density matrices for noisy Quantum circuits using IBM Qiskit, showing that Bayesian networks are a valid abstraction for Quantum correlated and uncorrelated noise events. The correctness of our results point to new languages and methods for representing Quantum Computations.

GOAL: Prove that probabilistic graphical models like Bayesian Networks are good abstractions for Noisy and Noise-Free Quantum Circuits.

Background (Quantum Computing)

- In classical computers, information is represented via bits, or physical systems that exist in only two possible states, commonly denoted by 0s and 1s.
- In quantum computers, information is represented via qubits, or quantum systems that can exist in a superposition of two classical states, $\alpha|0\rangle+\beta|1\rangle$, where $|\alpha|^2+|\beta|^2=1$.
- **Quantum Computing is Fast:** For certain computational problems, there exist algorithms on quantum computers that are more efficient than any known classical algorithms.
- **Quantum Computers are Noisy:** Due to interactions with the environment, disturbances in qubit states, called quantum noise/error events, can result in unexpected quantum circuit outputs. While quantum noise models exist for different types of error events, conventional noise models do not assume correlation between error events.

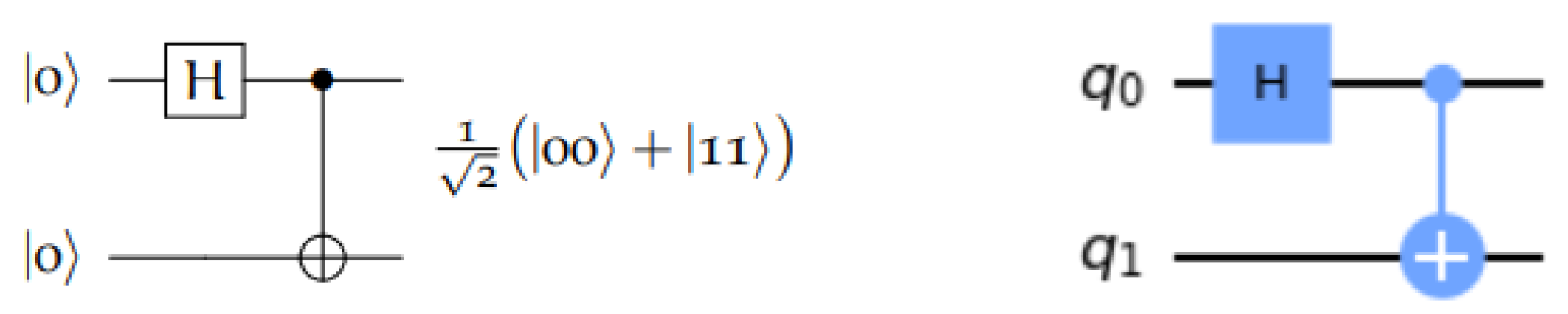


Figure 1: Bell State Circuit

Background (Bayesian Networks)

- Bayesian Networks are a mathematical model of how we represent causality for a list of random events. Formally, they are directed acyclic graphs.
- Nodes represent random events, and directed edges depict dependencies between random events. Each event contains an associated table of conditional probabilities, which describe probabilistically causality of event states between linked events.
- **Bayesian Networks are Visual:** The graphical structure of Bayesian Networks makes the modeling of correlated events easy to implement.
- **Bayesian Networks are Efficient:** Inference algorithms such as Variable Elimination allow for the quick generation of the marginal probability distribution for a list of queried output event states.

Methods and Materials

Algorithms

Firstly, we studied small-scale algorithms, and analyzed how we could map quantum algorithms to Bayesian Networks.

- In quantum Bayesian Networks, we represent qubit states at time moments with nodes, and quantum gates through conditional amplitude tables and edges.
- By allowing for complex numbers and altering normalization calculations, quantum circuits can be represented as quantum Bayesian Networks without loss of generality.

We modeled the following algorithms:

- **Bell State Circuit**
- **Deutsch-Jozsa Algorithm**
- **Simon's Algorithm**
- **Implementation of QFT gates**

Noise Models

We analyzed simple models for noise and error events in quantum circuits.

- **Noiseless Circuits:** Ideal quantum circuits whose output is a state vector.
- **Uncorrelated Noise:** Quantum circuits with a single probabilistic error event, whose output is a density matrix. Noise models include bit flip and amplitude damping.
- **Correlated Noise:** Quantum circuits with multiple dependent error events, whose output is a density matrix. Noise models include correlated bit flips.

Density matrices mathematically describe uncertainty in the output state due to the probabilistic nature of the error event.

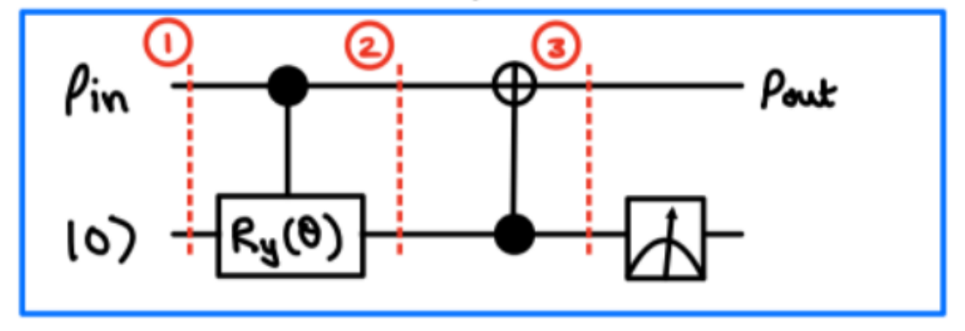


Figure 2: Amplitude Damping

Software

We made use of existing quantum and graphical software to facilitate our analysis of quantum Bayesian Networks.

- **IBM Qiskit:** Software for constructing and simulating simple quantum circuits.
- **Pgmpy:** Python Library for constructing and analyzing probabilistic graphical model, including Bayesian Networks.

To model quantum Bayesian Networks, we extended pgmpy to handle complex numbers and quantum normalization calculations.

- **Editing Source Code:** We altered several methods in pgmpy's source code to correctly handle quantum computing calculations.
- **New Functionality:** We created several functions to return pgmpy's output as familiar state vector and density matrix forms.

Results

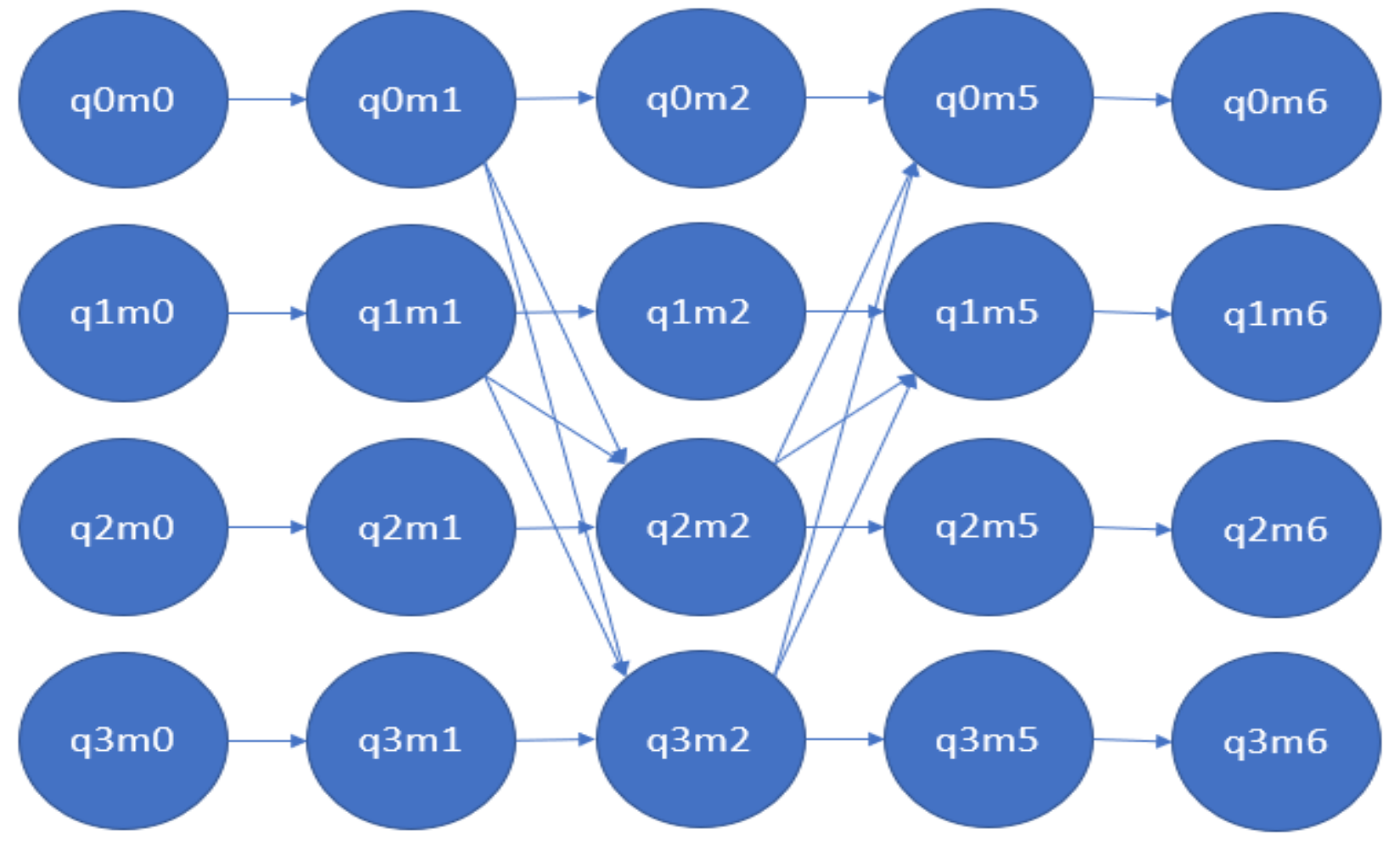


Figure 3: Simon's Algorithm Bayesian Network

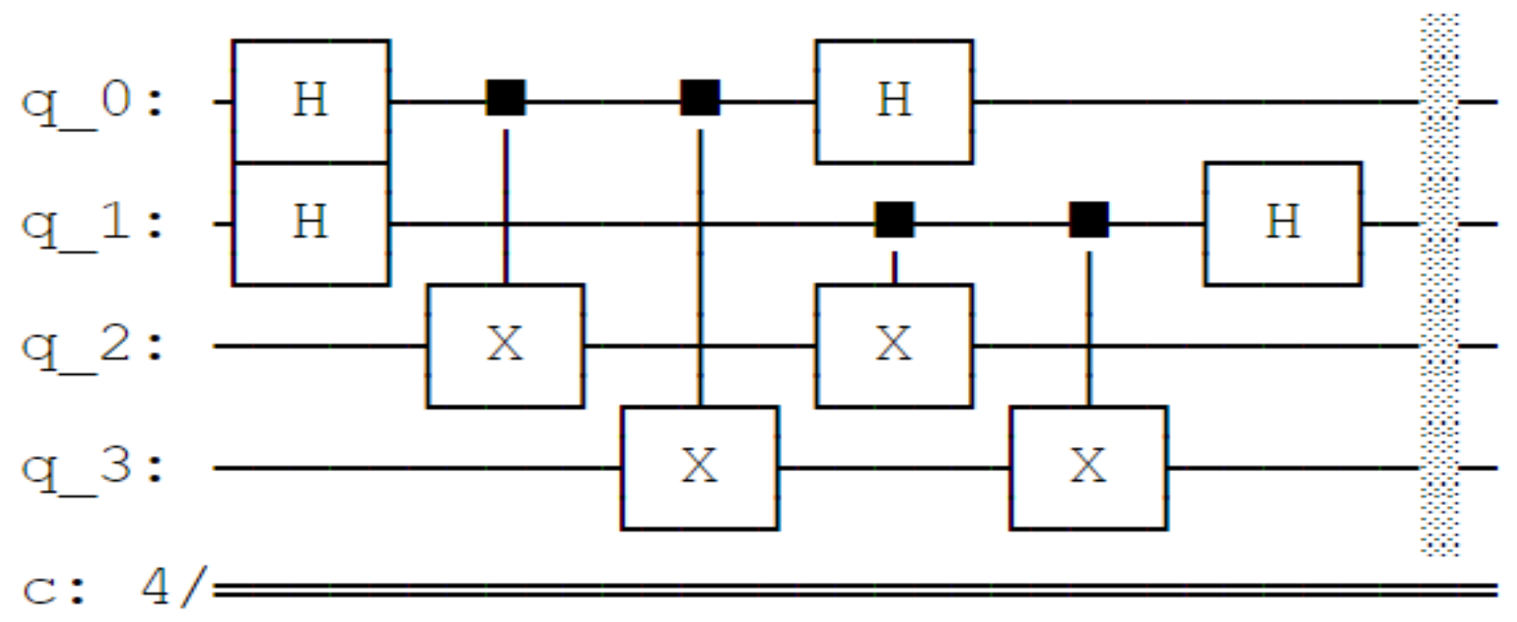
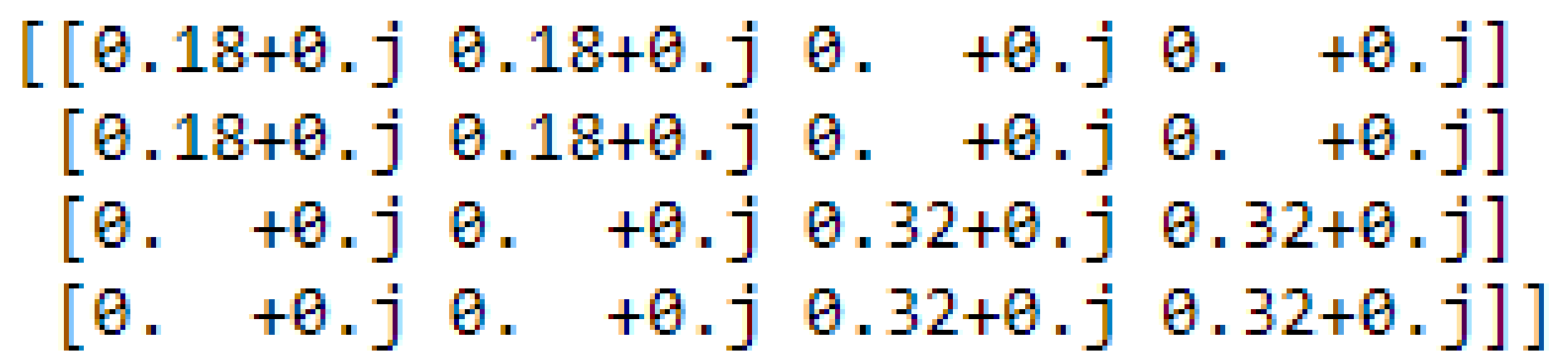


Figure 4: Simon's Algorithm Quantum Circuit

We tested our Bayesian Network representation of Quantum Circuits by taking a common Quantum Algorithm know as Simon's Algorithm and constructing it in both our updated version of pgmpy and Qiskit. In Figure 3, we can see the Bayesian Network abstraction of Simon's Algorithm and in Figure 4, we see the Quantum Circuit in Qiskit. We ran our simulations to produce Density Matrices that could be compared to test the validity of our simulation. On the right, we see an example output for our simulations. We tested the validity for Bit Flip events and Figure 5 shows the density matrix generated by pgmpy while Figure 6 shows the one generated by Qiskit. We notice that they both agree, thus proving our hypothesis.



```
[[0.18+0.j 0.18+0.j 0.   +0.j 0.   +0.j]
 [0.18+0.j 0.18+0.j 0.   +0.j 0.   +0.j]
 [0.   +0.j 0.   +0.j 0.32+0.j 0.32+0.j]
 [0.   +0.j 0.   +0.j 0.32+0.j 0.32+0.j]]
```

Figure 5: pgmpy Density Matrix for Bit Flip

State	Amplitude	State	Amplitude	State	Amplitude
0000	0.5	0110	0	0011	0.5
1000	0	1110	0	1011	0
0100	0	0001	0	0111	0
1100	0.5	1001	0	1111	-0.5
0010	0	0101	0		
1010	0	1101	0		

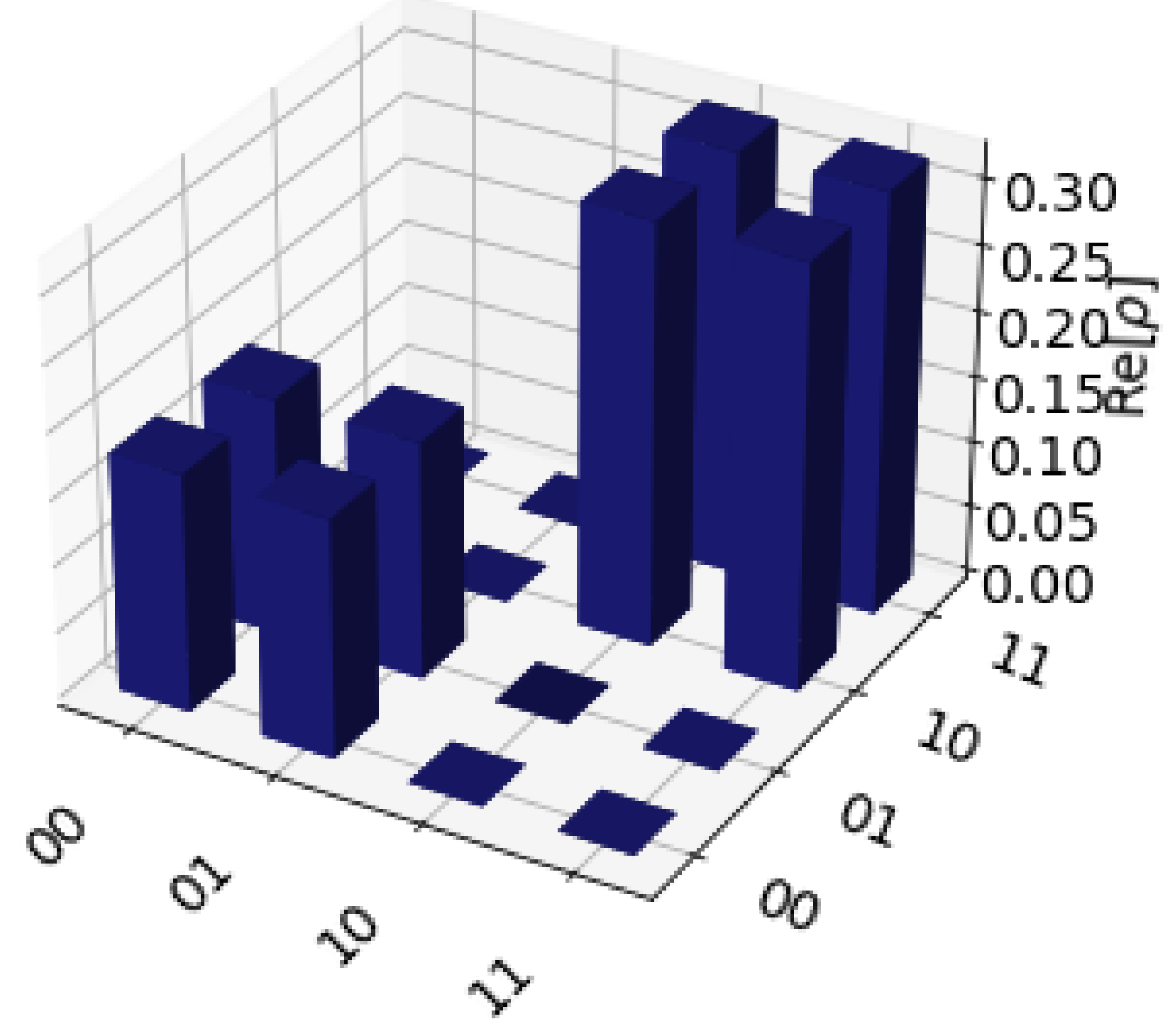


Figure 6: Qiskit Density Matrix for Bit Flip

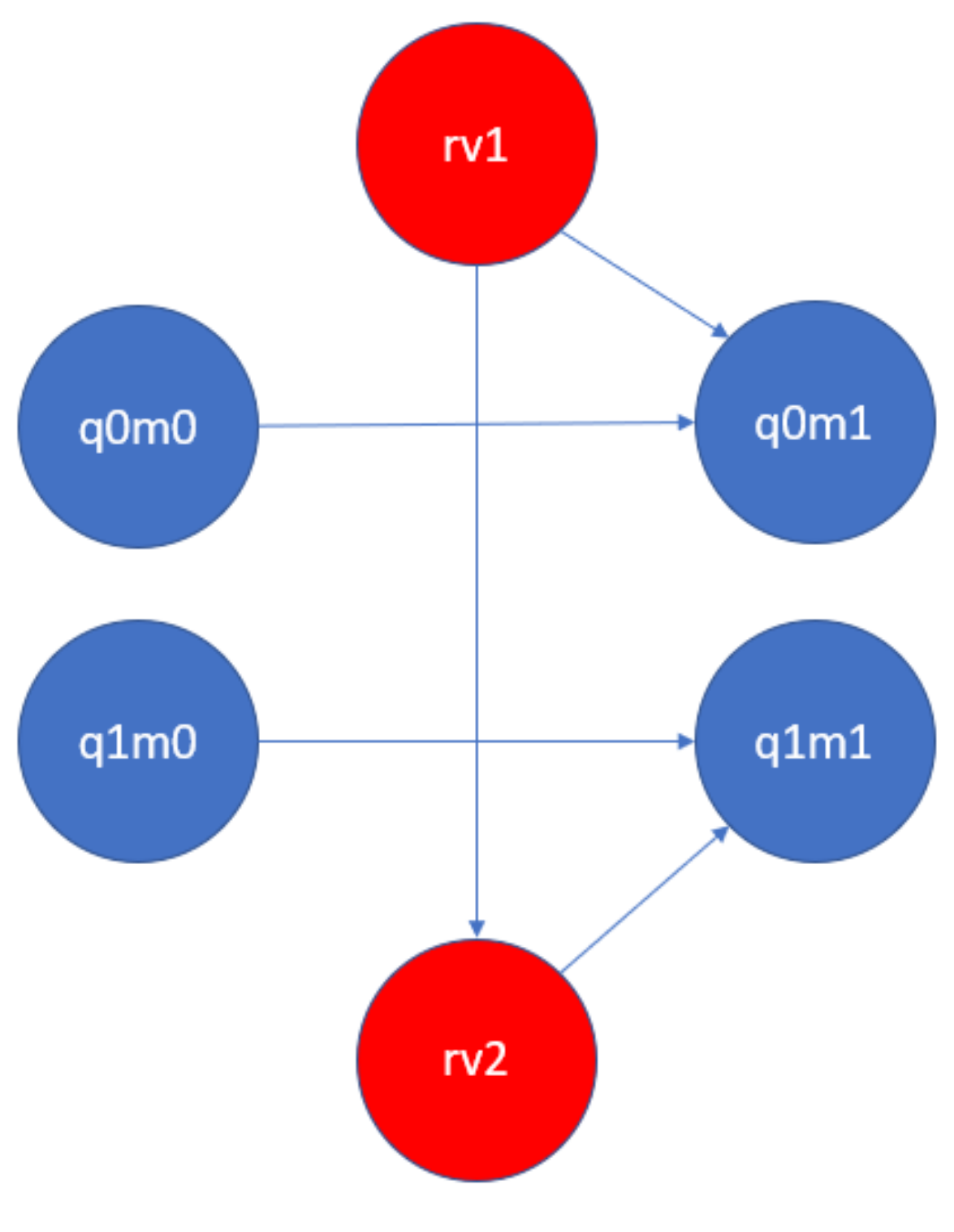


Figure 7: Correlated Bit Flip Bayesian Network

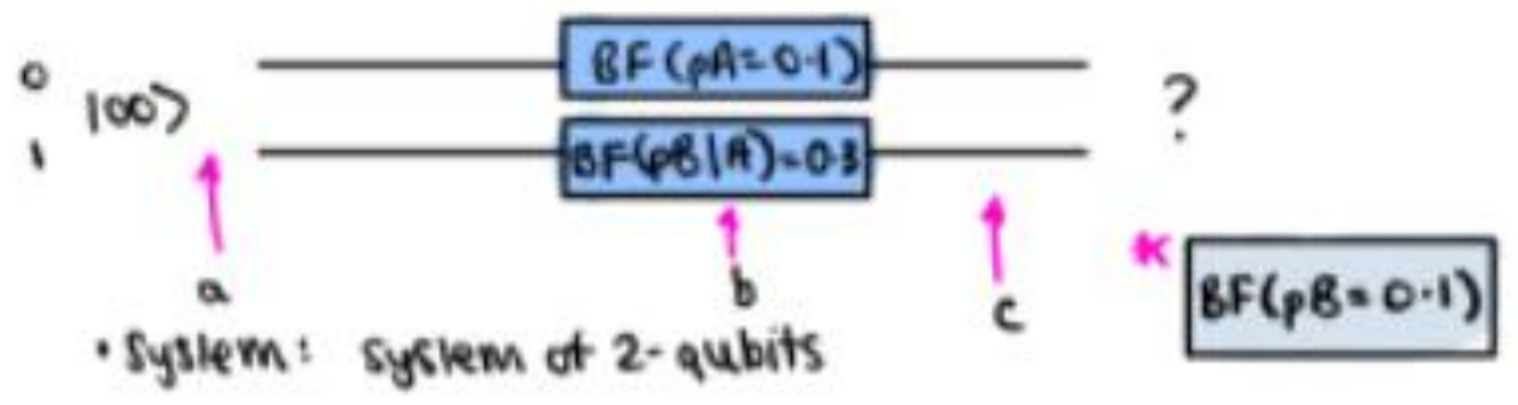
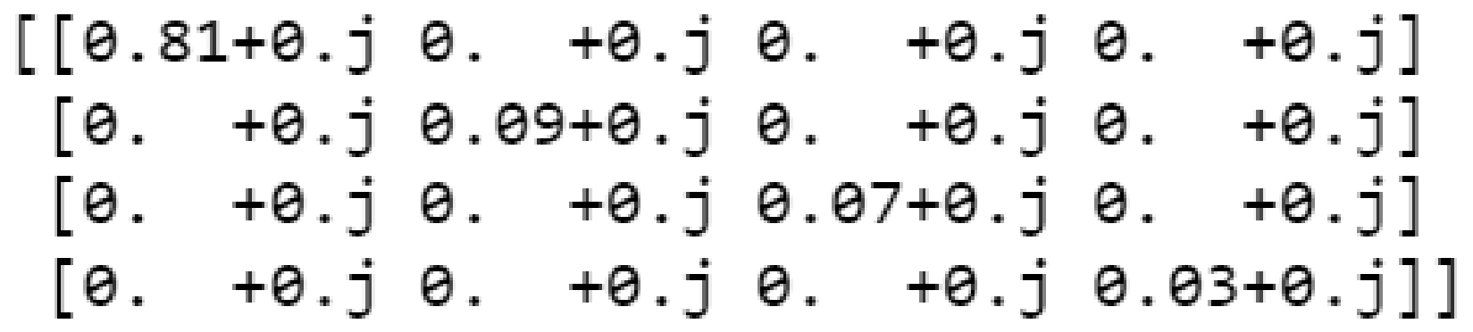


Figure 7: Correlated Bit Flip Circuit



```
[[0.81+0.j 0.   +0.j 0.   +0.j 0.   +0.j]
 [0.   +0.j 0.09+0.j 0.   +0.j 0.   +0.j]
 [0.   +0.j 0.   +0.j 0.07+0.j 0.   +0.j]
 [0.   +0.j 0.   +0.j 0.   +0.j 0.03+0.j]]
```

Figure 8: Correlated Bit Flip Density Matrix

After proving the validity of our abstraction for Noise Free Circuit events, we wanted to further extend our hypothesis and test the Bayesian Networks abstractions on events that Qiskit is unable to handle like Correlated Bit Flip events. We followed a similar testing process as before and created the Bayesian Network abstraction using pgmpy, and this time, we derived the theoretical circuit as well as its density matrix. Figure 8 shows the theoretical circuit and Figure 7 shows the Bayesian Network equivalent. We then compared the Density Matrices from our derivation and pgmpy and saw that their results aligned once again, proving that Bayesian Network abstractions are valid for both Noisy and Noise-Free circuits. Furthermore, they allow for simulation of Correlated Events that even Qiskit does not simulate.

Future Direction

- We aim to construct larger quantum Bayesian Networks with a more diverse set of noise models to analyze more practical algorithms in quantum computers, such as Shor's prime factoring algorithm.
- We plan to continue developing our pgmpy infrastructure, allowing for quantum software such as Qiskit to directly communicate with pgmpy. This will facilitate analysis of larger quantum circuits and complex noise models.
- In the long term, we hope to add our Quantum Model functionality to the pgmpy toolset allowing for other Quantum Researchers and Developers to simulate Bayesian Network abstractions of Quantum Circuits.

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